Probability - UNIT-2

1) State chebyshev's Inequality ? Discuss it applications.

If x is a random variable with $E(x) < \infty$. $V(x) < \infty$. for any +ve number k Then $P[|(x - E(x))| \geq k] \leq \frac{\sigma^2}{k^2}$

Applications:

The chebyshev's Inequality is used to find the consistent estimator.

It is used in defining conversions in probability and Almost sure.

It is used to solve the problem under CLT (central limit theorem) and LAM (Law of large number)

It can also used in CAN and BAN estimator

2) Define characteristics function. state its properties.

let x be the random variable then the characteristics function defined by

$$\phi_x(t) = E(e^{itx}) = \sum_{x=0}^{\infty} e^{itx} P(x = x_i)$$

$$= \int_x e^{itx} dF(x)$$

$$= \int_x e^{itx} f(x) dx$$

Properties

If 'x' is R.V with Pdf $f(x)$ and characteristic function as $\phi_x(t)$. let $y = ax + b$ then $\phi_y(t) = e^{ibt} \phi_x(at)$

If x is R.V with characteristic function $\phi_x(t)$ and $\mu_r' = E(x^r)$ exist then $\mu_r' = (-i)^r \left[\frac{\partial^r \phi_x(t)}{\partial t^r} \right]_{t=0}$

let 'x' be the R.V defined on (Ω, B, P) with the cumulative distribution $f(x)$ be the characteristic function $\phi_x(t)$ then $|\phi_x(t)| \leq 1$

$\phi_x(0) = 1$ and $\phi_x(-t) = \overline{\phi_x(t)}$

3) State levy's continuity theorm?

Every characteristic function is uniformedy continuous a the whole real time.

4) State and prove cauchy - schwartz inequality.

10 " long

6) state and prove the uniqueness theorm.

6 " long

7) Define ch. function show that $|\phi_x(t)| \leq 1$

consider $|\phi_x(t)| = \left|\int_{-\infty}^{\infty} e^{itx} \, dF(x)\right|$

$$\leq \int_{-\infty}^{\infty} |e^{itx}| \, dF(x)$$

$$\leq \int_{-\infty}^{\infty} 1 \, dF(x) \quad \left[\because e^{itx} = \cos tx + i \sin tx \right.$$

$$|e^{itx}| = \sqrt{\cos^2 tx + \sin^2 tx}$$

$$\left. = 1 \right]$$

$$\leq \int_{-\infty}^{\infty} f(x) \, dx$$

$\therefore |\phi_x(t)| \leq 1$

8) Markov Inequality

5 " long

9) If $\phi_x(t)$ is ch. function of a R.V 'x' then S.T

$1 - Re\,\phi(2t) \leq 4[1 - Re\,\phi(t)]$

consider $1 - Re\,\phi(2t) = 1 - E(\cos 2t)$

$$= 1 - \int_{-\infty}^{\infty} \cos 2tx \, dF(x)$$

$$= \int_{-\infty}^{\infty} f(x)\,dx - \int_{-\infty}^{\infty} \cos 2tx \, f(x)\,dx$$

$$= \int_{-\infty}^{\infty} (1 - \cos 2tx) f(x)\,dx$$

$$= \int_{-\infty}^{\infty} 2\sin^2(tx)\, f(x)\,dx \qquad [\because 1-\cos 2\theta = 2\sin^2\theta]$$

$$= 2\int_{-\infty}^{\infty} (1-\cos^2(tx))\, f(x)\,dx$$

$$= 2\int_{-\infty}^{\infty} (1-\cos tx)(1+\cos tx)\, f(x)\,dx$$

WKT the $\cos x$ lies b/w 0 and 1 hence $1+\cos x \le 2$

$$1-\text{Re}\,\phi(2t) \le 2\int_{-\infty}^{\infty} (1-\cos tx)(2)\, f(x)\,dx$$

$$\le 4\int_{-\infty}^{\infty} (1-\cos tx)\, f(x)\,dx$$

$$\le 4\,E(1-\cos tx)$$

$$\le 4[1-\text{Re}\,\phi(t)]$$

10) liapunov's inequality
& long

11) state and prove triangular inequality

let x and y be the two RV with $E(x)<\infty$, $E(y)<\infty$ and $E(x+y)<\infty$ then $[E(x+y)^2]^{1/2} \le [E(x)^2]^{1/2} + [E(y^2)]^{1/2}$

Proof:

consider $(x+y)^2 = (x+y)(x+y)$

$$(x+y)^2 = x(x+y) + y(x+y)$$

$$|x+y|^2 = |x|\,|x+y| + |y|\,|x+y|$$

$$E|x+y|^2 = E|x|\,E|x+y| + E|y|\,E|x+y|$$

By using cauchy's inequality we have

$$E(|x+y|^2) \le [E|x|^2\, E|x+y|^2]^{1/2} + [E|y|^2\, E|x+y|^2]^{1/2}$$

dividing on bothsides by $[E(x+y)^2]^{1/2}$

$$\frac{E|x+y|^2}{[E|x+y|^2]^{1/2}} \le \frac{[E|x|^2]^{1/2}[E|x+y|^2]^{1/2}}{[E|x+y|^2]^{1/2}} + \frac{[E|y|^2]^{1/2}[E|x+y|^2]^{1/2}}{[E|x+y|^2]^{1/2}}$$

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$$\left[E|x+y|^2\right]^{1-\frac{1}{2}} \leq \left[E|x|^2\right]^{1/2} + \left[E|y|^2\right]^{1/2}$$

$$\left[E|x+y|^2\right]^{1/2} \leq \left[E|x|^2\right]^{1/2} + \left[E|y|^2\right]^{1/2}$$

12) Define ch. function and prove that

i) $\phi_x(0) = 1$ ii) $\phi_x(t) \leq 1$ iii) $\phi_x(-t) = \overline{\phi_x(t)}$

i) $\phi_x(0) = [\phi_x(t)]_{t=0}$

$= [E(e^{itx})]_{t=0}$

$= [E[e^{i(0)x}]$

$= 1$

ii) $\phi_x(t) \leq 1$

7 in short

iii) consider $\phi_x(-t) = [E(e^{i(-t)x})]_{t=0}$

$= E[\cos(-tx) + i\sin(-tx)]_{t=0}$

$= E[\cos(tx) - i\sin tx]_{t=0}$

$= E[\cos tx]_{t=0} - i\,E[\sin(tx)]_{t=0}$

$E(\cos tx) - iE(\sin tx)$ which is the conjugate of

$E(\cos tx) + i\,E(\sin tx) = \overline{\phi_x(t)}$

$\therefore \phi_x(-t) = \overline{\phi_x(t)}$

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State and prove Minkowski's inequality

ii) find the distribution of x where its $\phi_x(t) = e^{-\frac{1}{2}t^2\sigma^2}$ using Inversion theorem.

i) Minkowski's inequality:

let x and y be two R.V such that the expectation of $E(|x|^p) < \infty$, $E(|y|^p) < \infty$ $E(|x+y|^p) < \infty$ then we have

$$(E(|x+y|^p))^{1/p} \le E(|x|^p)^{1/p} + E(|y|^p)^{1/p}$$

Proof:

let x & y be two R.V

$$|x+y|^p = |x+y| \cdot |x+y|^{p-1}$$

$$|x+y|^p = |x|\,|x+y|^{p-1} + |y| \cdot |x+y|^p$$

By taking expectation on both sides

$$E|x+y|^p = E|x|\, E|x+y|^{p-1} + E|y| \cdot E|x+y|^{p-1}$$

By applying holder's inequality to the each term on right side by taking $q = \frac{p}{p-1}$ we get

$$E|x+y|^p \le (E|x|^p)^{1/p} \left(E|x+y|^{q(p-1)}\right)^{1/q} + (E|y|^p)^{1/p}\left(E|x+y|^{q(p-1)}\right)^{1/q}$$

By the above eq with $E(|x+y|^p)^{1/q}$ on b.s

$$[E|x+y|^p]^{1-\frac{1}{q}} \le (E|x|^p)^{1/p} \cdot \frac{(E|x+y|^{q(p-1)})^{1/q}}{(E|x+y|^p)^{1/q}} + \frac{(E|y|^p)^{1/p}(E|x+y|^{q(p-1)})^{1/q}}{(E|x+y|^p)^{1/q}}$$

$$(E|x+y|^p)^{1-\frac{1}{q}} \le \frac{(E|x|^p)^{1/p}\, E|x+y|^{\frac{p}{p-1}(p-1)})^{1/q}}{(E|x+y|^p)^{1/q}} + \frac{(E|y|^p)^{1/p}(E|x+y|^{\frac{p}{p-1}(p-1)})^{1/q}}{(E|x+y|^p)^{1/q}}$$

$$\left[\because q = \frac{p}{p-1} ; \frac{1}{q} = \frac{p-1}{p} ; \frac{1}{q} = 1 - \frac{1}{p} ; \Rightarrow 1 - \frac{1}{q} = \frac{1}{p}\right]$$

$$\therefore (E|x+y|^p)^{1/p} \le (E|x^p|)^{1/p} + (E|y|^p)^{1/p}$$

Hence proved

5) i) State and prove Markov's Inequality?
ii) " " " Jensen's " ?

i) Markov's Inequality:

If x be the R.V define on pseudo space (Ω, B, P), takes non -ve values with $E(x) < \infty$ and $a > 0$ then

$$\text{prob} P[x > a] \leq \frac{E(x)}{a}$$

Proof:

let x be the continuous RV with PdF $f(x)$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{a} x \cdot F(x) dx + \int_{a}^{\infty} x \cdot F(x) dx$$

$$E(x) \geq \int_{a}^{\infty} x \cdot F(x) dx \qquad (\because x = a) \text{ (sub min value of } a)$$

$$\geq \int_{a}^{\infty} a f(x) dx \qquad \Rightarrow \frac{E(x)}{a} \geq P[x \geq a]$$

$$P(x \geq a] \leq E(x)/a$$

Hence proved.

ii) Jensen's Inequality:

let 'x' be the R.V and $f(x)$ be the convex function of x then $f(E(x)) \leq E(F(x))$

Proof:

let 'x' be the random variable whose value lies on I and 'f' be the convex function on I

A function is said to be convex for every $x_0 \in I$ then corresponding no $f(x_0)$ such that for every $x \in I$

$$\Rightarrow \lambda x_0 (x - x_0) \leq f(x) f(x_0)$$

Replacing x_0 with $E(x)$ we get

$$\lambda E(x) E(x - E(x)) \leq E(F(x)) - f(E(x))$$

$$\lambda E(x) (E(x) - E(x)) \leq E(f(x) - f(E(x)))$$

$$0 \leq E(f(x) - f(E(x)))$$

$$\therefore f(E(x)) \leq E(f(x))$$

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nsen's inequality is used to prove the variance of x

$V(x) \geq 0$

let $f(x) = x^2$

$$E(x^2) \geq (E(x))^2 \Rightarrow E(x^2) - (E(x))^2 \geq 0$$

$$V(x) \geq 0$$

6) i) State and prove uniqueness theorm of characteristic function

ii) " " " minkowsky's Inequality

i) The 2 distribution function $F(x) = \{F_1(x), F_2(x)\}$ are identical iff then the ch. function $\phi_1(t)$ & $\phi_2(t)$ are identical.

(or)

The distribution function $F(x)$ of a r.v and its ch.func determined each other

i.e., ch.fun uniquely determined the dist. function.

Proof:

let $F_1(x)$, $F_2(x)$ be the two distribution function corresponding to a given ch. function $\phi(t)$ then from the inversion theorm

for any $a < b$

we have $F_1(b) - F_2(b) = F_1(a) - F_2(a)$ to all the common points of continuety of F_1 and F_2.

from the inversion theorm

$$F_2(b) - F_2(a) = F_1(b) - F_1(a) = c$$

$$F_1(b) - F_2(b) = F_1(a) - F_2(a) = c$$

let us allow 'b' to vary for a fixed 'a'

and WKT $F_1(\infty) = 1$ $F_2(\infty) = 1$

$F_1(\infty) - F_2(\infty) = 0$ then we have

$$F_1(b) - F_2(b) = F_1(a) - F_2(a) = c$$

AS $b \to \infty$

$F_1(a) - F_2(a) = 0$

$F_1(a) = F_2(a)$

At all the continuous point of F_1 & F_2

$F_1 = F_2$

i.e., 'F' uniquely determined by $\phi_x(t)$

ii) long 4 (i)

8) State and prove Lyapanov Inequality.

Let X be the r.v and $(E|X|^b) < \infty$ then for orbitary a&b

$0 \leq a \leq b \quad [E|X|^a]^{1/a} \leq [E|X|^b]^{1/b}$

Proof:

Let $a \leq b$ then $\lambda = \frac{b}{a}$ which is ≥ 1

we can represent Jensen's inequality as

$$E|Y|^\lambda \geq [E|Y|]^\lambda \quad \ldots\ldots (1)$$

By sub $Y = |X|^a$ & $\lambda = b/a$ in eq (1)

$$[E|X|^a]^{b/a} \geq [E|X|^a]^{b/a}$$

$$E|X|^b \geq [E|X|^a]^{b/a}$$

by taking the power $1/b$ on b.s

$$(E|X|^b)^{1/b} \geq [E|X|^a]^{b/a * 1/b}$$

$$[E|X|^b]^{1/b} \geq [E|X|^a]^{1/a}$$

Hence proved

2) State and prove Holder's Inequality.

Let X & Y be the two r.v satify

$E[|X|^p] < \infty$, $E(|Y|^q) < \infty$ & $E(|XY|) < \infty$

$$E(|XY|) \leq [E(|X|^p)]^{1/p} \, [E|Y|^q]^{1/q}$$

Proof:

P and

satisfying

p and q be the two real +ve numbers satisfying, $1 < p, q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$

then $\frac{a^p}{p} + \frac{b^q}{q} \geq ab$ .... (1)

which is ~~Eur~~ cramer's Inequality

let $a = \frac{|x|}{[E(|x|^p)]^{1/p}}$ and $b = \frac{|y|}{[E|y|^q]^{1/q}}$

a and b value sub in eq (1)

$$\frac{\left(\frac{|x|}{[E|x|^p]^{1/p}}\right)^p}{p} + \frac{\left(\frac{|y|}{[E(|y|^q)]^{1/q}}\right)^q}{q} \geq \frac{|x|}{[E(|x|^p)]^{1/p}} \cdot \frac{|y|}{[E(|y|^q)]^{1/q}}$$

$$\frac{1}{p}\left[\frac{|x|}{[E(|x|^p)]^{1/p}}\right]^p + \frac{1}{q}\left[\frac{|y|}{[E|y|^q]^{1/q}}\right]^q \geq \frac{|x|}{[E(|x|^p)]^{1/p}} \cdot \frac{|y|}{[E|y|^q]^{1/q}}$$

$$\frac{1}{p} \cdot \frac{|x|^p}{E(|x|^p)} + \frac{1}{q} \frac{|y|^q}{E(|y|^q)} \geq \frac{|x|}{(E(|x|^p))^{1/p}} \cdot \frac{|y|}{[E(|y|^q)]^{1/q}}$$

$$[E(|x|^p)]^{1/p} [E(|y|^q)]^{1/q} \left[\frac{|x|^p}{p \cdot E|x|^p} + \frac{|y|^q}{q \cdot E|y|^q}\right] \geq |x||y|$$

$$[E(|x|^p)]^{1/p} (E(|y|^q))^{1/q} \left[\frac{E|x|^p}{p \cdot E|x|^p} + \frac{E|y|^q}{q \cdot E|y|^q}\right] \geq E[|x||y|]$$

$$[E|x|^p]^{1/p} [E|y|^q]^{1/q} \geq ~~Et~~ \; E|xy|$$

1) State and prove the Inversion theorem of characteristic function at continuty points x_1 & x_2 $(x_1 < x_2)$ of F.

Proof:

consider $\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-itx_1} - e^{-itx_2}}{it} \phi_x(t) dt$

$$= \frac{1}{2\pi}\int_{-T}^{T} \frac{e^{-itx_1} - e^{-itx_2}}{it} \int_{-\infty}^{\infty} e^{itx} f(x)\,dx \Big] dt$$

changing the order of integration, we have

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-T}^{T} \frac{e^{-it(x_1)} - e^{-itx_2}}{it} (e^{itx}) F(x)\,dx\right] dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-T}^{T} \frac{e^{-it(x-x_1)} - e^{-it(x-x_2)}}{it} dt\right] dx$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-T}^{T} \left[\frac{\cos t(x-x_1) + i\sin t(x-x_1) - \cos t(x-x_2) - i\sin t(x-x_1)}{t} dt\right] dF(x)\right.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-T}^{T} \frac{\sin t(x-x_1) - \sin t(x-x_2)}{t} dt\right] dF(x).$$

$$= \frac{2}{2\pi}\int_{-\infty}^{\infty}\left[\int_{0}^{T} \frac{\sin t(x-x_1)}{t} - \frac{\sin t(x-x_2)}{t} dt\right] dF(x)$$

$$= \frac{1}{\pi} \lim_{T\to\infty} \int_{-\infty}^{\infty}\left[\int_{0}^{\infty} \frac{\sin t(x-x_1)}{t} - \frac{\sin t(x-x_2)}{t} dt\right] dF(x)$$

let $I = \frac{1}{\pi}\int_{-\infty}^{\infty}\left[\frac{\sin t(x-x_1)}{t} - \frac{\sin t(x-x_2)}{t} dt\right]$ ..... (1)

$\rightarrow I = \frac{1}{\pi}\left[\frac{-\pi}{2} - \left(\frac{-\pi}{2}\right)\right] = 0$; $x < x_1 < x_2$

$\rightarrow I = \frac{1}{\pi}\left[0 - \left(\frac{-\pi}{2}\right)\right] = \frac{1}{\pi}\left[\frac{\pi}{2}\right] = \frac{1}{2}$; $x = x_1 < x_2$

$\rightarrow I = \frac{1}{\pi}\left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right] = 1$; $x_1 < x < x_2$

$\rightarrow I = \frac{1}{\pi}\left[\frac{\pi}{2} - \frac{\pi}{2}\right] \Rightarrow 0$ when $x_1 < x_2 < x$

from eq (1) & (2) we have

$$\text{RHS} = \int_{-\infty}^{\infty} I \cdot dF(x)$$

$$= \int_{x} 1 \cdot dF(x) \quad x_1 < x_2 < x$$

$$= \int_{x_1}^{x_2} dF(x) \Rightarrow F(x_2) - F(x_1)$$

$$F(x_2) - F(x_1) = \lim_{T\to\infty} \frac{1}{2\pi}\int_{-T}^{T} \frac{e^{-itx_1} - e^{-itx_2}}{it} \phi_x(t)\,dt$$

state

i) let

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state and prove cauchy - schwartz inequality

ii) state " " chebysuev's inequality.

i) let x and y be the two random variable defined on the probability space with $E(x^2) < \infty$ $E(y^2) < \infty$ $E(xy) < \infty$ then $[E(xy)]^2 \leq E(x^2) E(y^2)$ holds if y is linear function of x.

i.e., $y = tx$ with the probability one

Proof:

$$h(t) = E[(tx - y)^2]$$

$$= E[t^2x^2 + y^2 - 2txy]$$

$$= t^2 E(x^2) + E(y^2) - 2t E(xy)$$

The eq h(t) seens to be Quadratic function of t in the form of $at^2 + bt + c$

where $a = E(x^2)$.

$b = -2 E(xy)$

$c = E(y^2)$

$\Rightarrow$ as t is a real root if $b^2 - 4ac \geq 0$

as b value is negative $b^2 \leq 4ac$.

$$[2(-E(xy))]^2 \leq 4 E(x^2) E(y^2)$$

$$4[E(xy)]^2 \leq 4 E(x^2) E(y^2)$$

$$[E(xy)]^2 \leq E(x^2) E(y^2)$$

ii) If x is a r.v with $E(x) < \infty$ $V(x) < \infty$ for any number 'k' then the $[P[|x - E(x)| \geq k] \leq \frac{\sigma^2}{k^2}$

then

Proof:

let x be the continuous random variable with the pdf of $f(x)$.

let $k > 0$ define $\sigma^2 = E[(x - E(x))]^2$

$$= E[x - \mu]^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

$$\therefore \sigma^2 = \int_{-\infty}^{\mu - k\sigma} (x-\mu)^2 f(x) dx + \int_{\mu - k\sigma}^{\mu + k\sigma} (x-\mu)^2 f(x) dx + \int_{\mu + k\sigma}^{\infty} (x-\mu)^2 f(x) dx$$

$$= \int_{-\infty}^{\mu - k\sigma} (x-\mu)^2 f(x) dx + \int_{\mu + k\sigma}^{\infty} (x-\mu)^2 f(x) dx$$

We know that $x \leq \mu - k\sigma$ and $x \geq \mu + k\sigma$

$x - \mu \leq -k\sigma$ and $x - \mu \geq k\sigma$

which can be expressed as $|x - \mu| \geq k\sigma$

then $\sigma^2 \geq \int_{-\infty}^{\mu - k\sigma} k^2\sigma^2 f(x) dx + \int_{\mu + k\sigma}^{\infty} k^2\sigma^2 f(x) dx$

$$\sigma^2 \geq k^2\sigma^2 \left[\int_{-\infty}^{\mu - k\sigma} f(x) dx + \int_{\mu + k\sigma}^{\infty} f(x) dx \right]$$

$$\sigma^2 \geq k^2\sigma^2 [P(x \leq \mu - k\sigma) + P(x \geq \mu + k\sigma)]$$

$$\sigma^2 \geq k^2\sigma^2 [P(x - \mu) \leq -k\sigma) + P(x - \mu) \geq k\sigma]$$

$$\sigma^2 \geq k^2\sigma^2 [P(|x - \mu| \geq k\sigma)]$$

$$\frac{\sigma^2}{k^2\sigma^2} \geq P(|x - \mu| \geq k\sigma)$$

$\therefore$ $P[|x - \mu| \geq k\sigma] \leq \frac{1}{k^2}$ (or) $[P |x - \mu| \geq k] \leq \frac{\sigma^2}{k^2}$

$P[|x - \mu| \leq k] \geq 1 - \frac{\sigma^2}{k^2}$

the distribution function of a random variable is symmetric then show that its characteristic function is real?

symmetric property:

A distribution function is symmetric $\Leftrightarrow$ its ch. function is real and even (or) if a r.v 'x' symmetric about zero.

i.e., $f(-x) = f(x)$. Then the ch. function $\phi_x(t)$ is real and even, then 'x' is symmetric about zero.

Proof:

Necessary condition:

let $F(x)$ be the symmetric about the zero (or) $F(x)$ is a even function of x.

consider $\phi_x(t) = E[e^{itx}]$

$= E[\cos tx + i \sin tx]$

$= E(\cos tx) + i E(\sin tx)$

$= \int_{-\infty}^{\infty} \cos tx f(x) dx + i \int_{-\infty}^{\infty} \sin tx f(x) dx$

WKT $f(x)$ is a even function

$\cos(-x) = \cos x$, $\cos(tx)$ is an even function then

$\cos tx f(x)$ is also a even function

$\sin(-x) = -\sin x$, $\sin(tx)$ is an odd function

i.e., $\sin tx f(x)$ is also a odd function

$\phi_x(t) = \int_{-\infty}^{\infty} \cos tx f(x) dx + i(0)$

$\phi_x(t)$ is a real

WKT $\phi_x(t)$ is a real then $\phi_x(t)$ is even

Sufficient condition:

let $\phi_x(t)$ be real and even then we have to prove that $f(x)$ is symmetric about zero

let $y = -x$

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$$\phi_y(t) = \phi_{-x}(t)$$
$$= \overline{\phi_x(t)}$$
$$= \phi_x(t)$$

consider $F_Y(y) = P.(Y \leq y)$
$$= P(-x \leq y)$$
$$= P(x \geq -y)$$
$$= 1 - P(x \leq -y)$$
$$= 1 - F_x(-y)$$

Now differentiating on b.s w.r.to y

$$F_y(y) = 1 - F_x(-y)$$
$$F_y(y) = F_x(x) \qquad [\because y = -x]$$
$$F_{-x}(y) = F_x(x)$$

$\therefore$ $f(x)$ is symmetric about zero, when $\phi_x(t)$ is real and even.